Speed of flow of non-wetting droplets in capillaries of circular cross-section

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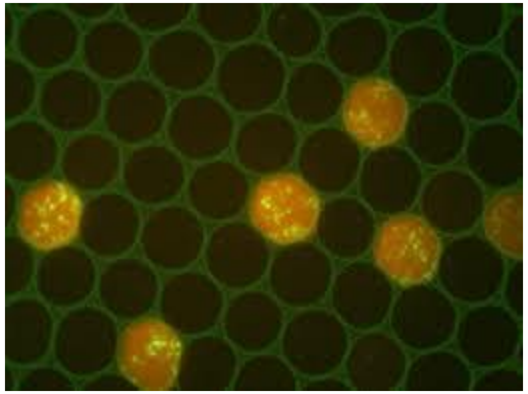
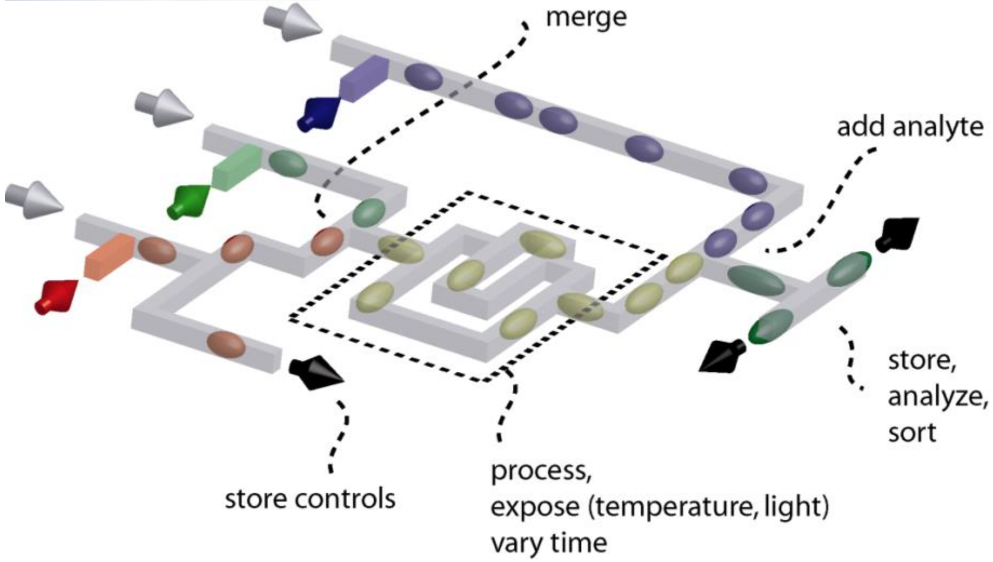
21.12.2016 IPPT PAN

Motivation

Compartmentalization:
-chemical reactions
-encapsulate and cultivate microorganisms

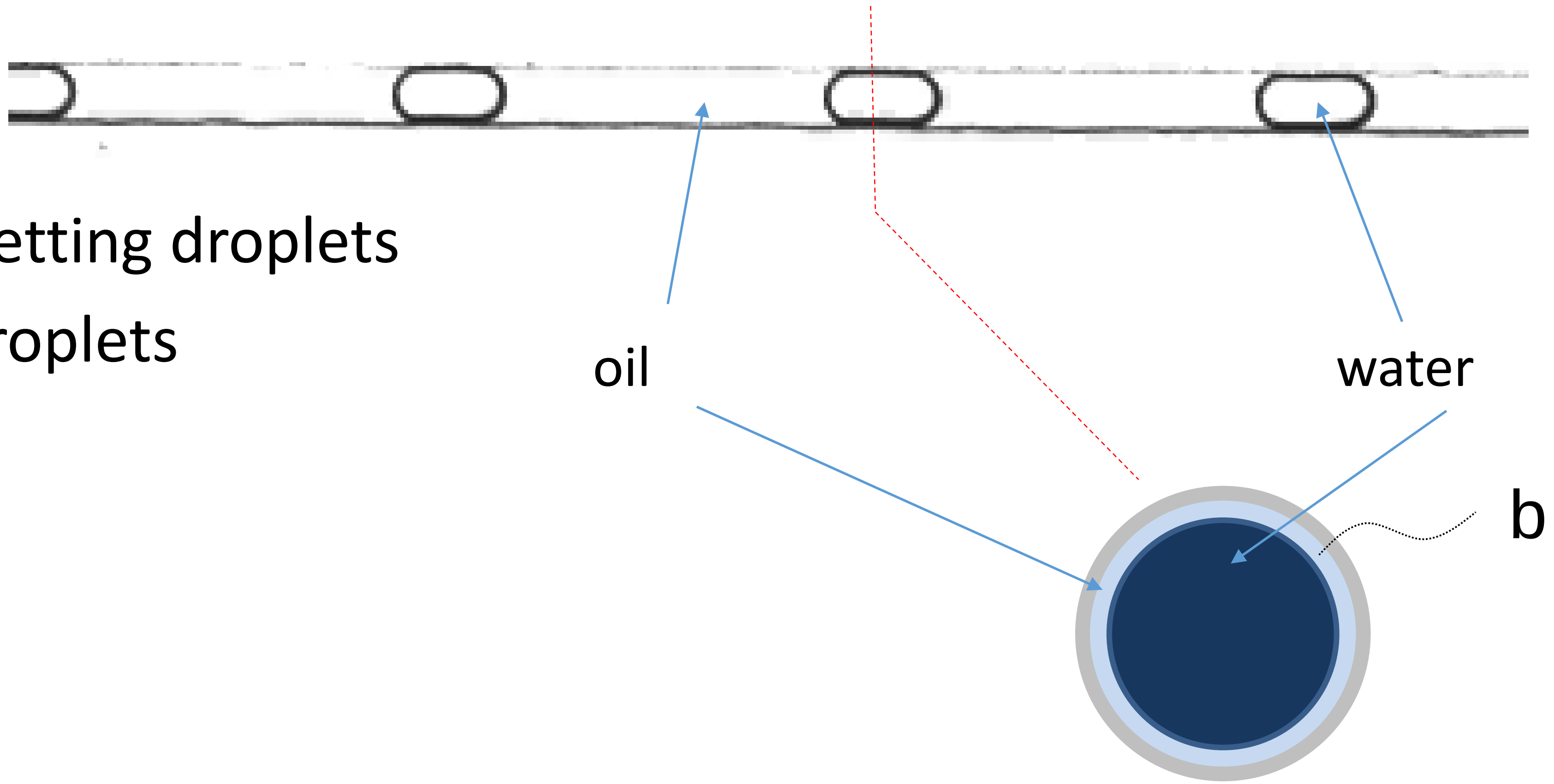




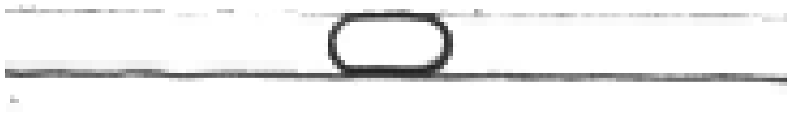
Goal

to understand speeds of flow of droplets in microfluidic channel

- non-wetting droplets
- long droplets



Goal: experiments on droplets and its theoretical explanation



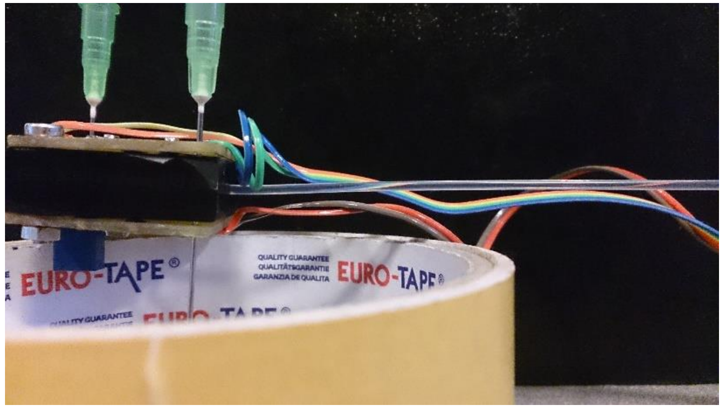
Mobility of droplet:

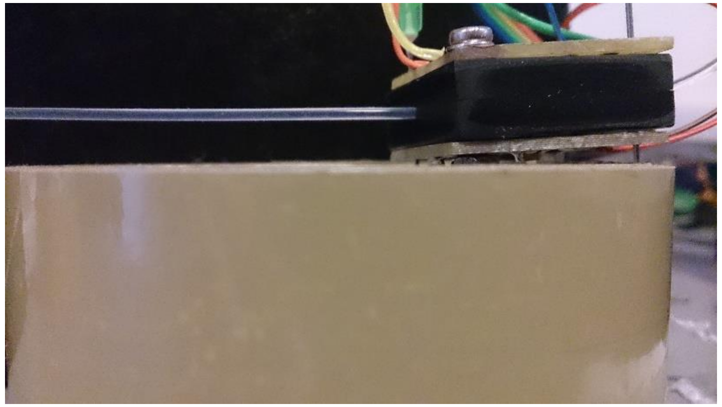
$$\beta = \frac{U}{V}$$

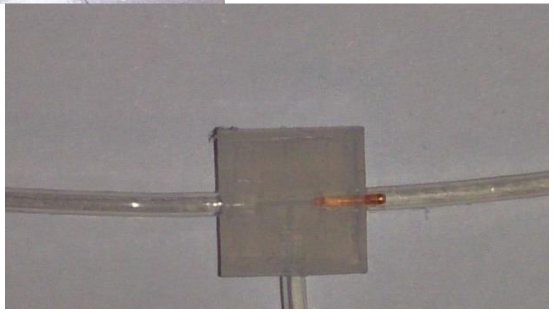
velocity of droplet

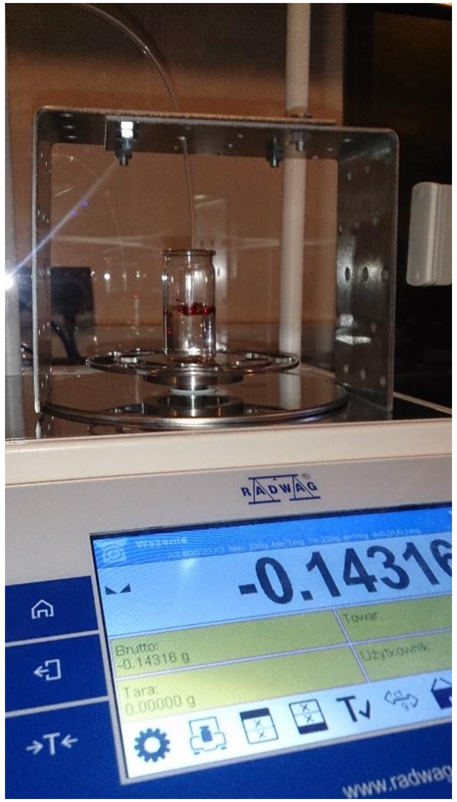
average velocity of continuous phase

Experimental setup









The same droplet travels back and forward

Measurement procedure – first approach

Procedure

-droplets travel back and forward

-distance between detectors

-time of passage of droplet between two detectors

$$U = \frac{L}{T}$$

-density of oil

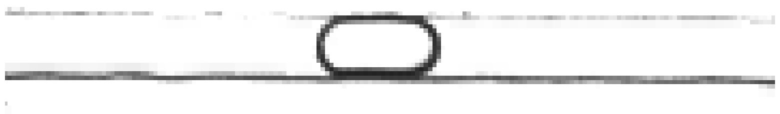
-tube diameter

-volumetric flow rate

$$V = \frac{4q}{\rho_{oil}\pi d^2}$$

$$\beta = \frac{U}{V}$$

Factors which influence speed of droplet



- average speed of flow of oil
- length of droplet
- viscosity of the droplet
- viscosity of the continuous liquid
- interfacial tension
- presence/absence of surfactant (oil, droplet)
- gravity field
- …

Dominant effects

water droplets in FC-40 oil
tube diameter: $d = 0.8\,mm$

Viscous and interfacial forces: $Ca = \frac{\mu_c V}{\sigma}$ ← ≈ 0.001 for $V = 1\,\frac{\text{cm}}{s}$

Inertial and viscous effects: $Re = \frac{\rho_c d V}{\mu_c}$ ← ≈ 3.6 for $V = 1\,\frac{\text{cm}}{s}$

Gravitational and interfacial effects: $Bo = \frac{\Delta\rho d^2}{\sigma} \approx 0.01$

Theoretical model: stationary Stokes equations

Governing parameters:

- Ratio of viscosities $\lambda = \frac{\mu_d}{\mu_c}$

- Capillary number $Ca = \frac{\mu_c V}{\sigma}$

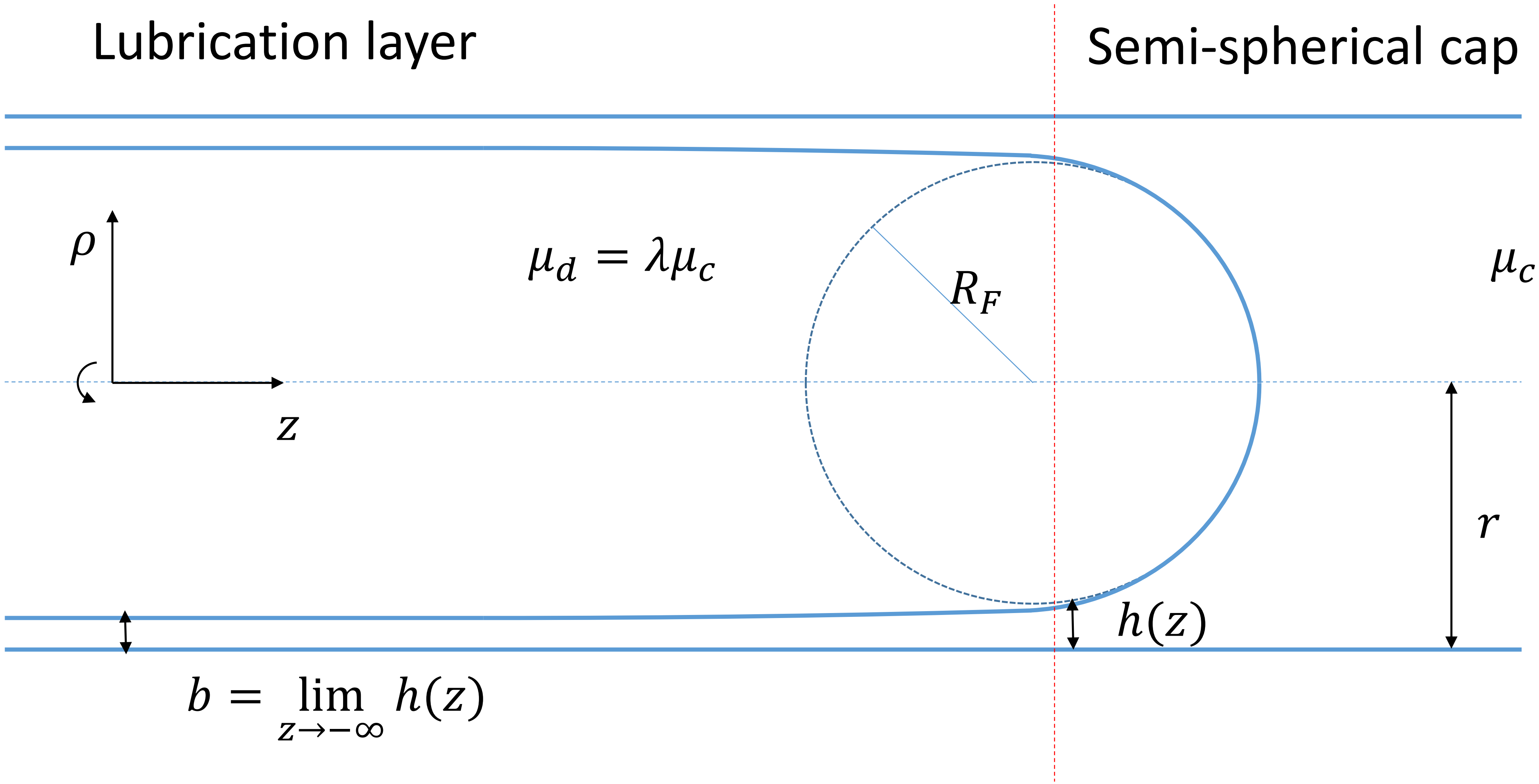
- Length of droplet l

  (numerical simulations: mobility does not change for $l > 2d$)

  tube diameter

Lac, E. & Sherwood, J., JFM, 2009, 640, 27-54

Theoretical model: stationary Stokes equations



Theoretical model: important results for droplets in circular cross-section channel

Bretherton (1961) – inviscid droplet ($\lambda = 0$), low capillary number

(Bretherton, F., JFM, 1961, 10, 166-188)

$$\beta = 1 + 1.29(3Ca)^{\frac{2}{3}} \qquad Ca = \frac{\mu_c V}{\sigma}$$

Extension of Bretherton's approach for viscous droplets:

Between parallel plates:

Teletzke, G. F.; Davis, H. T. & Scriven, L. Revue de Physique Appliquee, 1988, 23, 989-1007

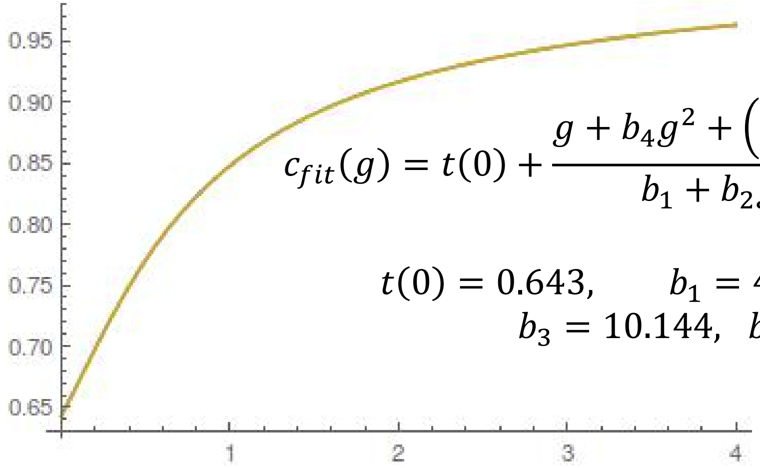
In channels of circular cross-sections:

Hodges, S.; Jensen, O. & Rallison, J., JFM, 2004, 501, 279-301

→ Numerical solution of ordinary (3rd order) differential equation leading to $\beta(\lambda, Ca)$

Our theoretical results

Consequent application of low Ca condition leads to scaling, and mobility of droplet ($\lambda = \frac{\mu_d}{\mu_c}$, $Ca = \frac{\mu_c V}{\sigma}$,) can be calculated as follows:



$$c_{fit}(g) = t(0) + \frac{g + b_4 g^2 + \left(t(0)2^{2/3} - t(0)\right) g^3}{b_1 + b_2 g + b_3 g^2 + g^3}$$

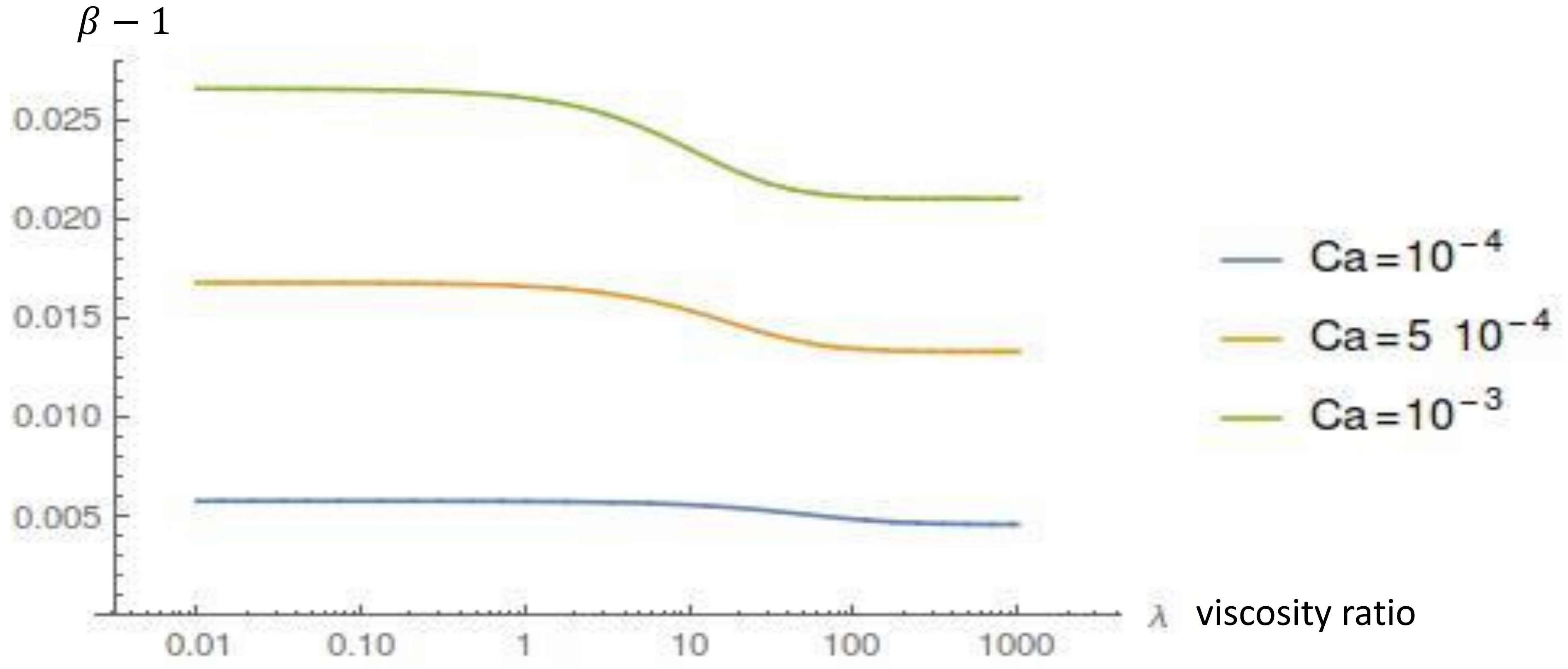
$t(0) = 0.643$, $b_1 = 4.109$, $b_2 = 8.906$, $b_3 = 10.144$, $b_4 = 3.575$.

Film thickness:

$$\frac{b}{r} = (3Ca)^{\frac{2}{3}} c\left(\lambda (3Ca)^{\frac{2}{3}}\right),$$

$$\beta \equiv \frac{U}{V} = 1 + 2\frac{b}{r}\frac{1 + 2\frac{b}{r}\lambda}{1 + 4\frac{b}{r}\lambda}.$$

Our theoretical results



→ To observe the above changes with viscosity, relative error of measurement of mobility should be less than about 1/1000

How to achieve relative error 1/1000 in experiment?

-distance between detectors
-time of passage of droplet between two detectors

$$U = \frac{L}{T}$$

-density of oil
-tube diameter
-volumetric flow rate

$$V = \frac{4q}{\rho_{oil}\pi d^2}$$

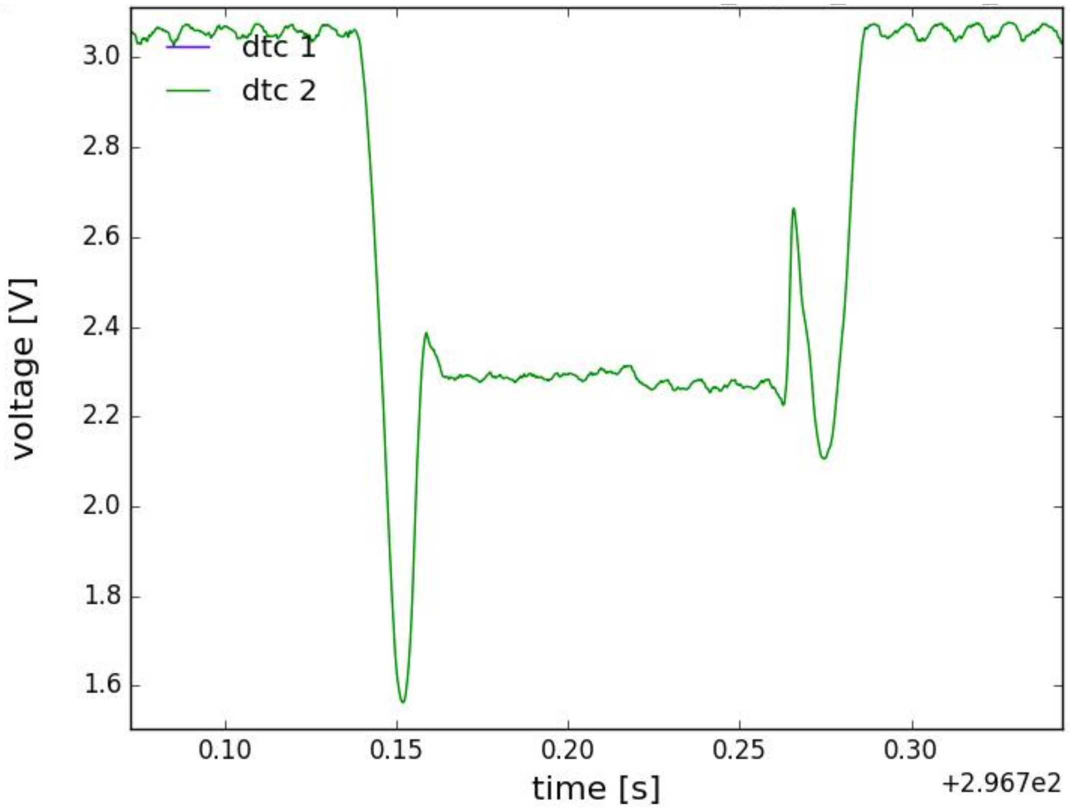
$$\beta = \frac{U}{V}$$

It is difficult to achieve high accuracy in this way…

How to achieve relative error 1/1000 in experiment?

Two features of our setup:

- time of passage T between two detectors (signal from detectors with frequency up to 5000Hz) $(T \sim seconds)$
- stability and proportionality of volumetric flow



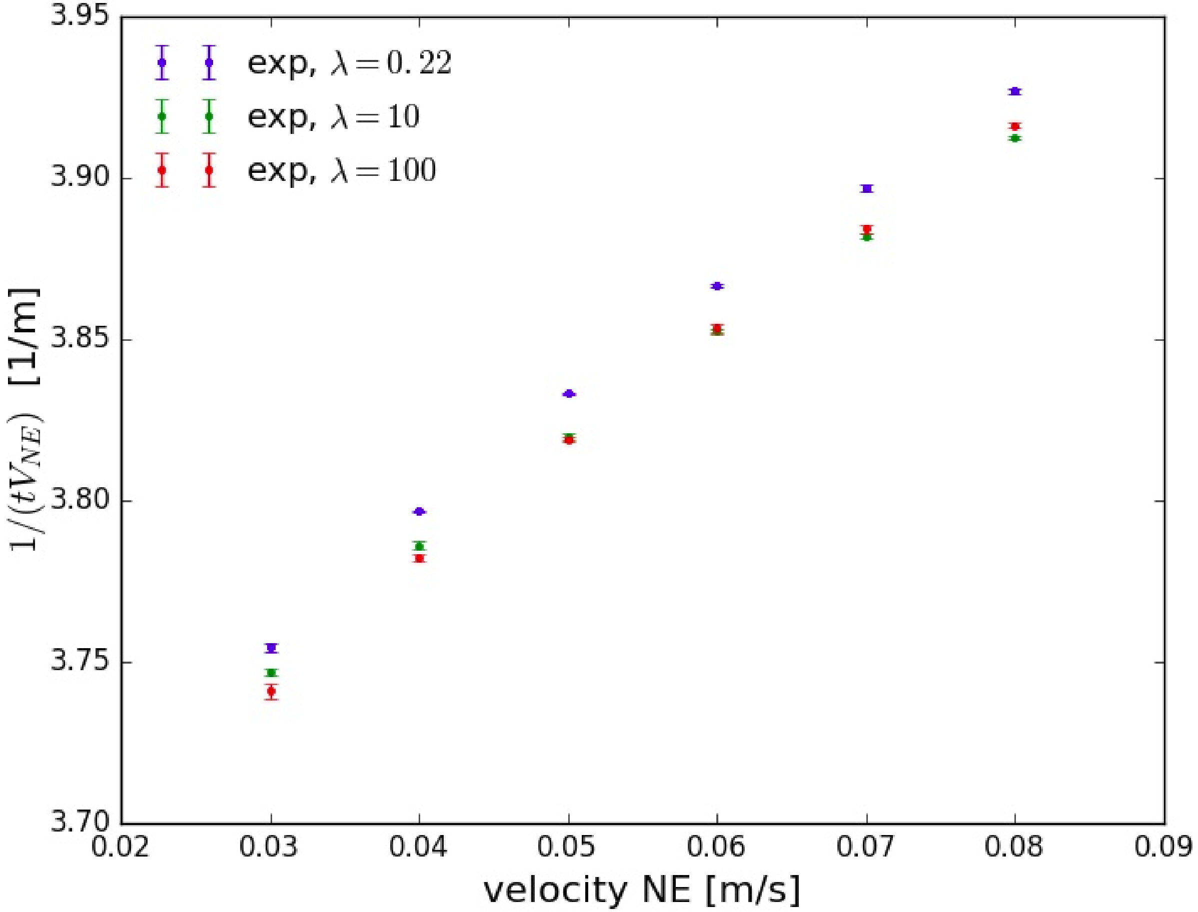
$$V = \alpha V_{NE}$$

Property of experimental setup (tube diameter, properties of pump/syringes) (droplet independent)

Measurements

For given droplet (water+glycerol+dye) in FC-40 oil (λ, σ) we set velocity on Nemesis pumps V_{NE} ,
and measure time of passage T

$$V_{NE}\ ,\ \ 1/TV_{NE}$$



Fit of theoretical curve

$$\frac{1}{TV_{NE}} \equiv \frac{L}{\alpha V_{NE}\left(1+2\frac{b}{r}\frac{1+2\frac{b}{r}\lambda}{1+4\frac{b}{r}\lambda.}\right)}$$

$$Ca = \frac{\mu_c \alpha V_{NE}}{\sigma}$$

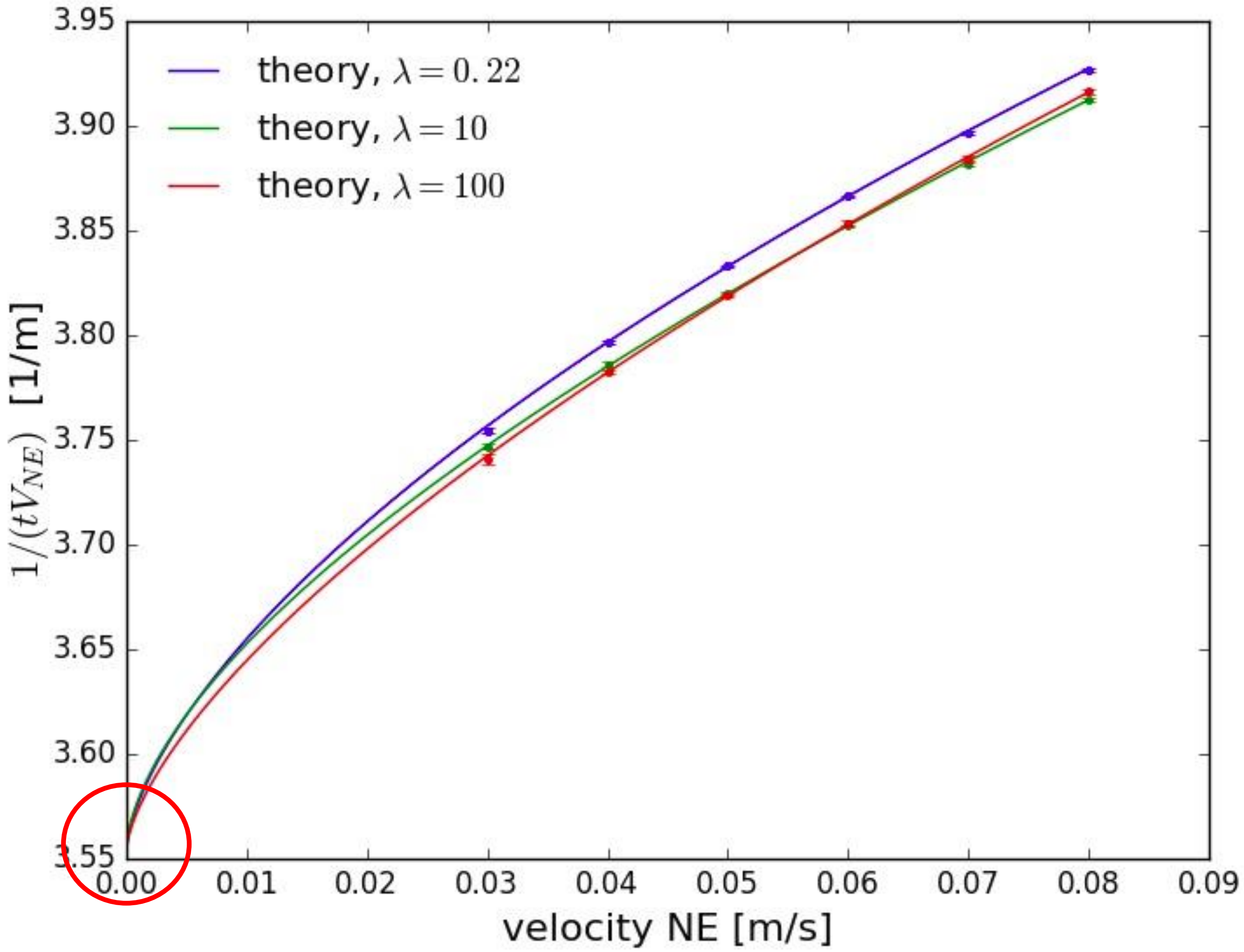
$$\frac{b}{r} = (3Ca)^{\frac{2}{3}} c\left(\lambda (3Ca)^{\frac{2}{3}}\right)$$

First test of the theory:

$\frac{L}{\alpha}, \lambda, \mu_c \alpha / \sigma$ as three independent fitting parameters

droplet independent

First test of the theory



Summary

- Theory: practical formula for mobility of long, non-wetting droplets
- Preliminary experimental verification (error of measurements < 1/1000)

Further goals:

- broader range of capillary numbers and viscosities
- can that be used to measure viscosity, surface tension? what accuracy?

In collaboration with Michał Horka, Jean Baptiste-Gorce and Piotr Garstecki